Math 31 – Summer 2013 **First Exam** Name Solutions

Instructions: You must show all work to receive full credit. There are 80 points possible on the exam.

Evaluate each of the following: (8 points each)

1. $\int x^2 \ln x\, dx =$

$u = \ln x \quad dv = x^2 dx$

$du = \frac{1}{x} dx \quad v = \frac{1}{3}x^3$

$\frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2 dx =$

$\boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$

2. $\int \frac{x-9}{x^2+3x-10} dx$

$\frac{x-9}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$

$x - 9 = A(x-2) + B(x+5)$

$x = -5: \quad -14 = -7A, \quad A = 2$

$x = 2: \quad -7 = 7B, \quad B = -1$

$\int \frac{x-9}{x^2+3x-10} dx = \int \left(\frac{2}{x+5} - \frac{1}{x-2}\right) dx$

$= \boxed{2\ln|x+5| - \ln|x-2| + C}$

3. $\int x^2 e^{-x^3} dx =$

Let $u = x^3$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$\frac{1}{3}\int e^{-u} du =$

$-\frac{1}{3}e^{-u} + C = \boxed{-\frac{1}{3}e^{-x^3} + C}$

4. $\int \sin^3(\pi x)\cos(\pi x)dx =$

Let $w = \pi x$

$dw = \pi dx$

$\frac{1}{\pi} dw = dx$

$\frac{1}{\pi}\int \sin^3 w \cos w \, dw$, Let $u = \sin w$, $du = \cos w \, dw$

$= \frac{1}{\pi}\int u^3 du = \frac{1}{4\pi} u^4 + c$

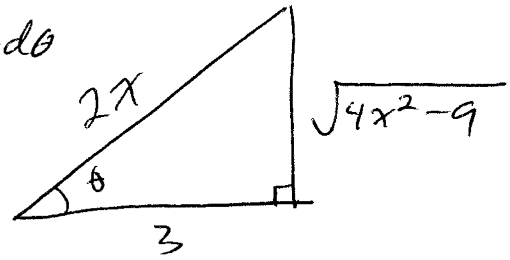
$= \boxed{\frac{1}{4\pi} \sin^4(\pi x) + C}$

5. $\int \frac{dx}{x^2\sqrt{4x^2-9}} =$

$x = \frac{3}{2}\sec\theta$

$dx = \frac{3}{2}\sec\theta\tan\theta \, d\theta$

$\sqrt{4x^2-9} = 3\tan\theta$



$\int \frac{\frac{3}{2}\sec\theta\tan\theta \, d\theta}{(\frac{3}{2}\sec\theta)^2(3\tan\theta)}$

$= \frac{2}{9}\int \frac{1}{\sec\theta} d\theta = \frac{2}{9}\int \cos\theta \, d\theta$

$= \frac{2}{9}\sin\theta + c$

$= \frac{2}{9} \cdot \frac{\sqrt{4x^2-9}}{2x} + c$

$= \frac{\sqrt{4x^2-9}}{9x} + c$

6. Evaluate $\int \cos\sqrt{x}\, dx$ by first making a substitution, and then using integration by parts.

Let $w = \sqrt{x}$, $dw = \frac{1}{2\sqrt{x}}dx$, or $dx = 2w\,dw$, then

$\int \cos\sqrt{x}\, dx = 2\int w\cos w \, dw$. Let $u = w$, $du = dw$, $dv = \cos w \, dw$, $v = \sin w$

$= 2\left[w\sin w - \int \sin w \, dw\right]$

$= 2\left[w\sin w + \cos w\right] + c$

$= 2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + C$

7. Evaluate $\int_0^\infty xe^{-2x}dx$ $= \lim_{t\to\infty} \int_0^t xe^{-2x}dx =$ (8 points)

$u = x \quad dv = e^{-2x}dx$

$du = dx \quad v = -\frac{1}{2}e^{-2x}$

$\lim_{t\to\infty}\left[-\frac{1}{2}xe^{-2x}\Big|_0^t + \frac{1}{2}\int_0^t e^{-2x}dx\right] =$

$\lim_{t\to\infty}\left[-\frac{1}{2}(te^{-2t} - 0) - \frac{1}{4}(e^{-2t} - 1)\right] =$

$-\frac{1}{2}\lim_{t\to\infty} te^{-2t} + 0 - \frac{1}{4}\lim_{t\to\infty} e^{-2t} + \frac{1}{4} = \boxed{\frac{1}{4}}$

8. Determine whether the improper integral $\int_0^{\pi/2} sec^2\, x\, dx$ converges. If it does, evaluate the integral. (Show all limits!) (8 points)

$\int_0^{\pi/2} \sec^2 x\, dx = \lim_{t\to\frac{\pi}{2}^-} \int_0^t \sec^2 x\, dx = \lim_{t\to\frac{\pi}{2}^-} \left[\tan x\right]_0^t =$

$\lim_{t\to\frac{\pi}{2}^-} \left[\tan t - \tan 0\right] = \infty$, so the improper integral diverges

9. For a function that is increasing and concave down on the interval [0, 2], four approximations have been obtained, each involving four subintervals. These are left-hand and right-hand Riemann sums, L_4 and R_4, and sums for the trapezoid rule T_4 and midpoint rule M_4. The estimates are as follows: 0.7614, 0.8648, 0.8702, and 0.9450. (4 points each)

a. Match the rule with the estimate.

$f'(x) > 0$

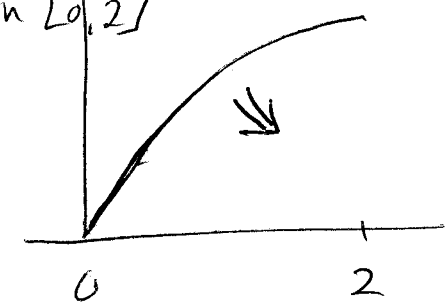
$f''(x) < 0$ on $[0, 2]$

$L_4 = 0.7614$

$R_4 = 0.9450$

$T_4 = 0.8648$

$M_4 = 0.8702$



b. Between what two values does $\int_0^2 f(x)\,dx$ lie?

$0.8648 < \int_0^2 f(x)dx < 0.8702$

10. Use the midpoint rule and trapezoidal rules, with $n = 5$, to estimate $\int_1^2 \frac{1}{x}\,dx$. (8 points)

$$\Delta x = \frac{2-1}{5} = \frac{1}{5}$$

$$M_5 = \Delta x\left[f(1.1) + f(1.3) + f(1.5) + f(1.7) + f(1.9)\right]$$

$$= \boxed{\frac{1}{5}\left[\frac{1}{1.1} + \frac{1}{1.3} + \frac{1}{1.5} + \frac{1}{1.7} + \frac{1}{1.9}\right]}$$

$$\approx 0.691908$$

$$T_4 = \frac{\Delta x}{2}\left[f(1) + 2f(1.2) + 2f(1.4) + 2f(1.6) + 2f(1.8) + f(2)\right]$$

$$= \boxed{\frac{1/5}{2}\left[\frac{1}{1} + \frac{2}{1.2} + \frac{2}{1.4} + \frac{2}{1.6} + \frac{2}{1.8} + \frac{1}{2}\right]}$$

$$\approx 0.695635$$